Domain and Asymptotes of Rational Functions

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Equations: (1) $f(x) = \dfrac{4x^2+3x+2}{4x-1}$ (2) $f(x) = \dfrac{2x}{x-2}$ (3) $f(x) = \dfrac{2x}{\sqrt{x-5}}$

Note: *The above numbered equations will be referenced below using the same notation e.g.* (1)

Domain depends on numerator and denominator

> We begin with the real numbers interval $(-\infty, \infty)$ then look for numbers or intervals to exclude:
> **1)** Numbers that cause only the denominator to be zero
> **2)** Numbers that cause an even root to be negative(whether its on the denominator or numerator)
> **3)** Numbers that cause a root of the denominator to be zero

Example: (3) Look at the denominator: $\sqrt{x-5} = 0$ when $x = 5$. Observe that $x < 5$ causes $(x-5) < 0$. Values under the even root must be positive therefore domain $= (5, \infty)$

Vertical, Horizontal, and Oblique Asymptotes

Vertical Asymptote: (1) Set the denominator equal to 0. You get the following:
$f(x) = 4x - 1 = 0 \Leftrightarrow 4x = 1 \Leftrightarrow x = \frac{1}{4}$
By plugging in x $= \frac{1}{4}$ for the numerator, you see that the numerator does not equal zero.
The above step is needed to ensure that x $= \frac{1}{4}$ is not a removable discontinuity.
Therefore, x=$\frac{1}{4}$ is a vertical asymptote.

Oblique Asymptote: (1) Divide the numerator by the denominator if the highest degree of the numerator is greater than the highest degree of the denominator.

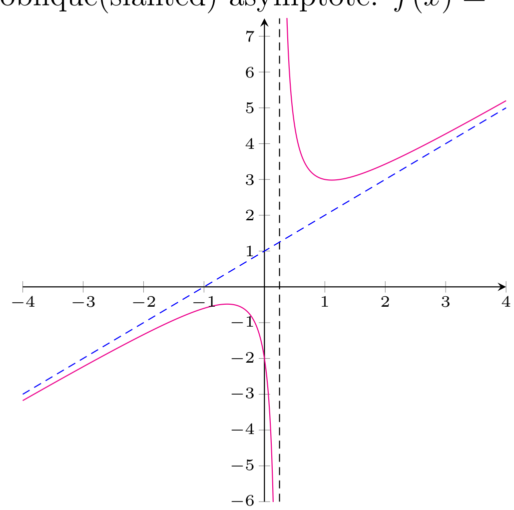
$$f(x) = \frac{8x^2+6x+4}{8x-2} = \frac{2(4x^2+3x+2)}{2(4x-1)}$$

Long division of the above equation is shown below. Division stops when the remainder's highest degree is less than the divisor. In this case, 8x-2 is the divisor. The oblique asymptote is y = x+1.

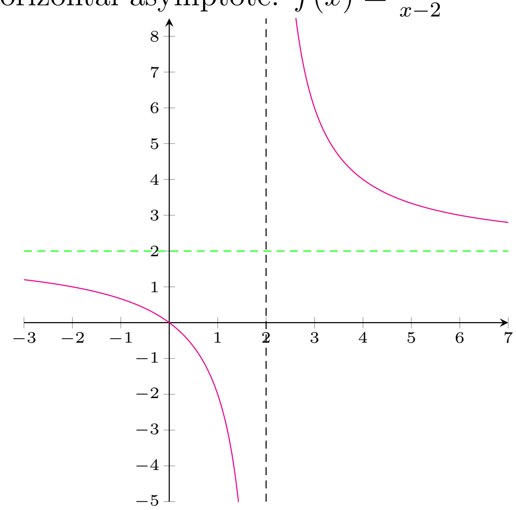
Horizontal Asymptote: The limit method may be used to find all horizontal asymptotes. (1) The degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote.
(2) Taking the limit as the x of numerator and denominator head to positive and negative infinity, we get that y = 2.

$$\begin{array}{r|r} & x+1 \\ \hline 8x-2 & 8x^2+6x+4 \\ & -\,8x^2+2x \\ \hline & 8x+4 \\ & -\,8x+2 \\ \hline & 6 \end{array}$$

The following function has a vertical and oblique(slanted) asymptote: $f(x) = \frac{8x^2+6x+4}{8x-2}$



The following function has a vertical and horizontal asymptote: $f(x) = \frac{2x}{x-2}$



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