California State University
SAN MARCOS

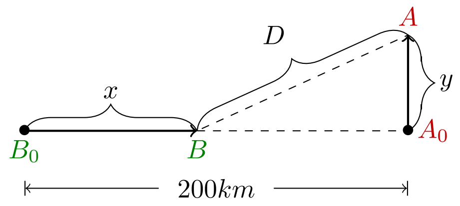
Calculus

Related Rates

Problem 1: Sailing Ships

At 1:00 PM ship A is 200 km east of ship B. Ship A is sailing north at 40 km/h and ship B is sailing east at 30 km/h. How fast is the distance between the ships changing at 4:00 PM?

1 Draw a picture and label your variables:



2 Create an equation (Pythagorean Theorem) based on the given data and your picture. Also, identify the rates:

$$D^2 = (200 \quad x)^2 + y^2$$
$$\frac{dx}{dt} = 30km/h$$
$$\frac{dy}{dt} = 40km/h$$
$$\frac{dD}{dt} = ? \text{ (This is what we want to find!)}$$

3 Take the derivative of the equation created in step 2 with respect to time:

$$D^2 = (200 \quad x)^2 + y^2$$
$$2D \cdot \frac{dD}{dt} = 2(200 \quad x) \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$
$$D \cdot \frac{dD}{dt} = (200 \quad x) \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt}$$

4 Find the distance that ship A traveled, y, and the distance that ship B traveled, x:

At t=3:

$$x = 30 \cdot 3 = 90km$$
$$y = 40 \cdot 3 = 120km$$

5 Use x and y to find D (the distance between the ships):

$$D^2 = (200 \quad 90)^2 + 120^2$$
$$D^2 = 12,100 + 14,400$$
$$D^2 = 26,500$$
$$D = \sqrt{26,500}$$

6 Plug in x, y and D and solve for $\frac{dD}{dt}$:

$$\sqrt{26,500} \cdot \frac{dD}{dt} = (200 \quad 90)(35) + 120(40)$$
$$\sqrt{26,500} \cdot \frac{dD}{dt} = 3,850 + 4,800$$
$$\sqrt{26,500} \cdot \frac{dD}{dt} = 950$$
$$\frac{dD}{dt} = \frac{950}{\sqrt{26,500}}$$
$$\frac{dD}{dt} \approx 5.84km/h$$

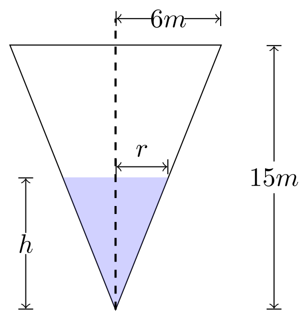
Problem 2: Volume A.K.A. The Leaking Cone

A cone shaped tank is filled with water but is leaking at a constant rate of $2m^3/h$. The base radius of the tank is $6m$ and the height is $15m$.

(a) How fast is the height of the water changing when the water is at $5m$?

(b) How fast is the radius changing when the height of the water is at $5m$?

1(a) Draw a picture and label your variables:



2(a) Create an equation (Volume of a cone) based on the given data and your picture, and identify rates:

$$V = \frac{1}{3}\pi r^2 h$$

The radius, r can be obtained using similar triangles

$$\frac{r}{h} = \frac{6}{15} \implies r = \frac{6}{15}h$$
$$V = \frac{1}{3}\pi(\frac{6}{15}h)^2 h$$
$$V = \frac{12}{225}\pi h^3$$

$$\frac{dV}{dt} = 2m^3/h$$
$$\frac{dr}{dt} = \text{rate for radius}$$
$$\frac{dh}{dt} = \text{rate for height}$$

3(a) Take the derivative of the equation created in step 2 with respect to time:

$$V = \frac{12}{225}\pi h^3$$
$$V = 3\left(\frac{12}{225}\right)\pi h^2$$
$$\frac{dV}{dt} = \frac{36}{225}h^2\frac{dh}{dt}$$

4(a) Plug in h and $\frac{dV}{dt}$, and solve for $\frac{dh}{dt}$:

$$2 = \frac{36}{225}(5)^2\frac{dh}{dt}$$
$$2 = 4\frac{dh}{dt}$$
$$\frac{1}{2} = \frac{dh}{dt}$$

This means that the height of the water is changing at a rate of $\frac{1}{2}m/h$

Now, for part **(b)**, we get to use information from part **(a)** and the ratio we used in step 2(b) to find $\frac{dr}{dt}$.

1(b) Take the derivative of the equation solved for r from similar triangles with respect to time:

$$r = \frac{6}{15}h$$
$$\frac{dr}{dt} = \frac{6}{15}\cdot\frac{dh}{dt}$$

2(b) Plug in $\frac{dh}{dt}$ from part (a) and solve:

$$\frac{dr}{dt} = \frac{6}{15}\left(\frac{1}{2}\right)$$
$$\frac{dr}{dt} = \frac{1}{5}$$

This means that the radius of the water is changing at a rate of $\frac{1}{5}m/h$