California State University SAN MARCOS

CALCULUS CONVERGENCE AND DIVERGENCE

TEST NAME	SERIES	CONVERGES	DIVERGES	ADDITIONAL INFO
nth TERM TEST	$\sum_{n=1}^{\infty} a_n$		if $\lim_{n\to\infty} a_n \neq 0$	One should perform this test first for divergence.
GEOMETRIC SERIES TEST	$\sum_{n=1}^{\infty} a_n r^{n \quad 1}$	if $\quad 1 < r < 1$	if $\lvert r \rvert \quad 1$	If convergent, converges to $s_n = \frac{a}{1 \quad r}$
P-SERIES TEST	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	if $p > 1$	if $p \leq 1$	Can be used for comparison tests.
INTEGRAL TEST	$\sum_{n=1}^{\infty} f(x)$	if $\int_1^{\infty} f(x) \cdot dx$ converges.	if $\int_1^{\infty} f(x) \cdot dx$ diverges.	$f(x)$ has to be continuous, positive, decreasing on $[1, \infty)$.
DIRECT COMPARISON TEST	$\sum_{n=1}^{\infty} a_n$	if $0 \leq a_n \leq b_n$, and $\sum_{n=1}^{\infty} b_n$ converges.	if $0 \leq b_n \leq a_n$, and $\sum_{n=1}^{\infty} b_n$ diverges.	For convergence, find a larger convergent series. For divergence, find a smaller divergent series.
LIMIT COMPARISON TEST	$\sum_{n=1}^{\infty} a_n$	if $\sum_{n=1}^{\infty} b_n$ converges, and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$.	if $\sum_{n=1}^{\infty} b_n$ diverges, and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$.	If necessary, apply L'Hospital's Rule. Inconclusive if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ or ∞.
ALTERNATING SERIES TEST	$\sum_{n=1}^{\infty} (\quad 1)^{n+1} a_n$	if $a_{n+1} \leq a_n$, and $\lim_{n\to\infty} a_n = 0$.	if $\lim_{n\to\infty} a_n \neq 0$.	To prove convergence prove that the sequence is decreasing and its limit is zero.
RATIO TEST	$\sum_{n=1}^{\infty} a_n$	if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$.	if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$.	The test fails if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$.
ROOT TEST	$\sum_{n=1}^{\infty} a_n$	if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$.	if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$.	The test fails if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$.

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DEFINITION OF CONVERGENCE AND DIVERGENCE

An infinite series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots$ is **convergent** if the sequence $\{s_n\}$ of partial sums, where each partial sum is denoted as $s_n = \sum_{n=1}^{n} a_n = a_1 + a_2 + \ldots + a_n$, is convergent.
If the sequence $\{s_n\}$ is divergent, then the series is called **divergent**.

ABSOLUTELY CONVERGENT

A series $\sum a_n$ is called **absolutely convergent** if the series of the absolute values $\sum |a_n|$ is convergent.

CONDITIONALLY CONVERGENT

A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.

$\sum_{n=1}^{\infty} c \cdot a_n = c \cdot \sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$	$\sum_{n=1}^{\infty} (a_n \quad b_n) = \sum_{n=1}^{\infty} a_n \quad \sum_{n=1}^{\infty} b_n$

POWER SERIES

A **power series** is a series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$ where x is a variable and the c_n's are called the **coefficients** of the series.

A series of the form $\sum_{n=0}^{\infty} c_n (x \quad a)^n = c_0 + c_1 (x \quad a) + c_2 (x \quad a)^2 + \ldots$ is called a **power series in (x a)** or a **power series centered at a** or a **power series about a**.

For a given power series $\sum_{n=0}^{\infty} c_n (x \quad a)^n$ there are only three possibilities:

(i) The series converges only when $x = a$.

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x \quad a| < R$ and diverges if $|x \quad a| > R$.

$$\frac{1}{1 \quad x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^{\infty} x^n \quad |x| < 1$$

If the power series $\sum_{n=0}^{\infty} c_n (x \quad a)^n$ has radius of convergence $R > 0$, then the function defined by $f(x) = \sum_{n=0}^{\infty} c_n (x \quad a)^n$ is differentiable on the interval $(a \quad R, a + R)$ and

(i) $f'(x) = \sum_{n=0}^{\infty} n c_n (x \quad a)^{n \quad 1}$.

(ii) $\int f(x) = C + \sum_{n=0}^{\infty} c_n \frac{(x \quad a)^{n+1}}{n+1}$.