California State University SAN MARCOS

# CALCULUS
## TRIGONOMETRIC DERIVATIVES AND INTEGRALS

### TRIGONOMETRIC DERIVATIVES

| | | |
|---|---|---|
| $\frac{d}{dx}(\sin(x)) = \cos(x)\cdot x'$ | $\frac{d}{dx}(\cos(x)) = \quad \sin(x)\cdot x'$ | $\frac{d}{dx}(\tan(x)) = \sec^2(x)\cdot x'$ |
| $\frac{d}{dx}(\csc(x)) = \quad \csc(x)\cot(x)\cdot x'$ | $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)\cdot x'$ | $\frac{d}{dx}(\cot(x)) = \quad \csc^2(x)\cdot x'$ |
| $\frac{d}{dx}(\sin^{\ 1}(x)) = \frac{1}{\sqrt{1 \quad x^2}}\cdot x'$ | $\frac{d}{dx}(\cos^{\ 1}(x)) = \quad \frac{1}{\sqrt{1 \quad x^2}}\cdot x'$ | $\frac{d}{dx}(\tan^{\ 1}(x)) = \frac{1}{1+x^2}\cdot x'$ |
| $\frac{d}{dx}(\csc^{\ 1}(x)) = \quad \frac{1}{x\sqrt{x^2 \quad 1}}\cdot x'$ | $\frac{d}{dx}(\sec^{\ 1}(x)) = \frac{1}{x\sqrt{x^2 \quad 1}}\cdot x'$ | $\frac{d}{dx}(\cot^{\ 1}(x)) = \quad \frac{1}{1+x^2}\cdot x'$ |

### TRIGONOMETRIC INTEGRALS

| | |
|---|---|
| $\int \sin(x)dx = \quad \cos(x) + C$ | $\int \csc(x)dx = \ln\lvert\csc(x) \quad \cot(x)\rvert + C$ |
| $\int \cos(x)dx = \sin(x) + C$ | $\int \sec(x)dx = \ln\lvert\sec(x) + \tan(x)\rvert + C$ |
| $\int \tan(x)dx = \ln\lvert\sec(x)\rvert + C$ | $\int \cot(x)dx = \ln\lvert\sin(x)\rvert + C$ |

| POWER REDUCTION FORMULAS | INVERSE TRIG INTEGRALS |
|---|---|
| $\int \sin^n(x) = \quad \frac{1}{n}\sin^{n \quad 1}(x)\cos(x) + \frac{n \quad 1}{n}\int \sin^{n \quad 2}(x)dx$ | $\int \sin^{\ 1}(x)dx = x\sin^{\ 1}(x) + \sqrt{1 \quad x^2} + C$ |
| $\int \cos^n(x) = \frac{1}{n}\cos^{n \quad 1}(x)\sin(x) + \frac{n \quad 1}{n}\int \cos^{n \quad 2}(x)dx$ | $\int \cos^{\ 1}(x)dx = x\cos^{\ 1}(x) \quad \sqrt{1 \quad x^2} + C$ |
| $\int \tan^n(x) = \frac{1}{n \quad 1}\tan^{n \quad 1}(x) \quad \int \tan^{n \quad 2}(x)dx$ | $\int \tan^{\ 1}(x)dx = x\tan^{\ 1}(x) \quad \frac{1}{2}\ln(1+x^2) + C$ |
| $\int \cot^n(x) = \quad \frac{1}{n \quad 1}\cot^{n \quad 1}(x) \quad \int \cot^{n \quad 2}(x)dx$ | $\int \frac{dx}{\sqrt{a^2 \quad x^2}} = \sin^{\ 1} \ \frac{x}{a} \ + C$ |
| $\int \sec^n(x) = \frac{1}{n \quad 1}\tan(x)\sec^{n \quad 2}(x) + \frac{n \quad 2}{n \quad 1}\int \sec^{n \quad 2}(x)dx$ | $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{\ 1} \ \frac{x}{a} \ + C$ |
| $\int \csc^n(x) = \quad \frac{1}{n \quad 1}\cot(x)\csc^{n \quad 2}(x) + \frac{n \quad 2}{n \quad 1}\int \csc^{n \quad 2}(x)dx$ | $\int \frac{dx}{x\sqrt{x^2 \quad a^2}} = \frac{1}{a}\sec^{\ 1} \ \frac{x}{a} \ + C$ |

## STRATEGY FOR EVALUATING $\int \sin^m(x)\cos^n(x)dx$

(a) If the power $n$ of cosine is odd ($n = 2k+1$), save one cosine factor and use $\cos^2(x) = 1 \quad \sin^2(x)$ to express the rest of the factors in terms of sine:

$$\int \sin^m(x)\cos^n(x)dx = \int \sin^m(x)\cos^{2k+1}(x)dx = \int \sin^m(x)(\cos^2(x))^k\cos(x)dx$$
$$= \int \sin^m(x)(1 \quad \sin^2(x))^k\cos(x)dx$$

Then solve by $u$-substitution and let $u = \sin(x)$.

(b) If the power $m$ of sine is odd ($m = 2k+1$), save one sine factor and use $\sin^2(x) = 1 \quad \cos^2(x)$ to express the rest of the factors in terms of cosine:

$$\int \sin^m(x)\cos^n(x)dx = \int \sin^{2k+1}(x)\cos^n(x)dx = \int (\sin^2(x))^k\cos^n(x)\sin(x)dx$$
$$= \int (1 \quad \cos^2(x))^k\cos^n(x)\sin(x)dx$$

Then solve by $u$-substitution and let $u = \cos(x)$.

(b) If both powers $m$ and $n$ are even, use the half-angle identities:

$$\sin^2(x) = \frac{1}{2}(1 \quad \cos(2x)) \qquad\qquad \cos^2(x) = \frac{1}{2}(1+\cos(2x))$$

## STRATEGY FOR EVALUATING $\int \tan^m(x)\sec^n(x)dx$

(a) If the power $n$ of secant is even ($n = 2k, k \quad 2$), save one $\sec^2(x)$ factor and use $\sec^2(x) = 1+\tan^2(x)$ to express the rest of the factors in terms of tangent:

$$\int \tan^m(x)\sec^n(x)dx = \int \tan^m(x)\sec^{2k}(x)dx = \int \tan^m(x)(\sec^2)^{k \quad 1}\sec^2(x)(x)dx$$
$$= \int \tan^m(x)(1+\tan^2(x))^{k \quad 1}\sec^2(x)(x)dx$$

Then solve by $u$-substitution and let $u = \tan(x)$.

(b) If the power $m$ of tangent is odd ($m = 2k+1$), save one $\sec(x)\tan(x)$ factor and use $\tan^2(x) = \sec^2(x) \quad 1$ to express the rest of the factors in terms of secant:

$$\int \tan^m(x)\sec^n(x)dx = \int \tan^{2k+1}(x)\sec^n(x)dx = \int (\tan^2(x))^k\sec^{n \quad 1}(x)\sec(x)\tan(x)dx$$
$$= \int (\sec^2(x) \quad 1)^k\sec^{n \quad 1}(x)\sec(x)\tan(x)dx$$

Then solve by $u$-substitution and let $u = \sec(x)$.

csusm.edu/stemsc XXX @csusm_stemcenter
Tel:
STEM SC (N): (760) 750-4101
STEM SC (S): (760) 750-7324