Math 160 Chapter 5/ Section 1-5: Riemann Sum, Indefinite Integrals, and Definite integrals Worksheet

Write the equation of the following:

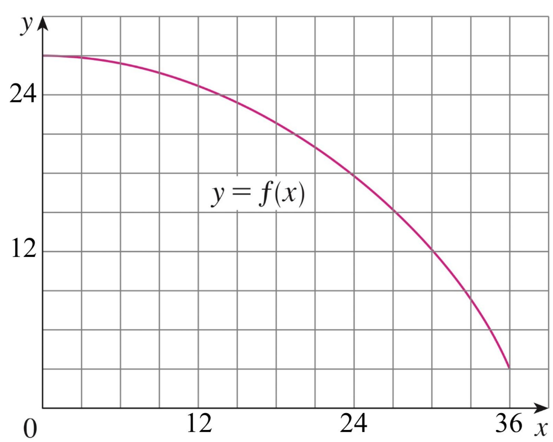
1. Definition of the area under a curve:

2. Definition of a definite integral:

3. Midpoint Rule:

Find the area under the curve:

1. Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 36$.



i. L_6 (sample points are left endpoints). Is this an over or underestimate of the true area?

ii. R_6 (sample points are right endpoints). Is this an over or underestimate of the true area?

iii. M_6 (sample points are midpoints). Is this an over or underestimate of the true area?

2. Estimate the area under the curve of $f(x) = x^2 - 1$ from $x = 1$ to $x = 5$ using 4 approximating rectangles and use left and right endpoints.

3. Estimate the area under the curve of $f(x) = 4\,sin(x)$ from $x = 0$ to $x = \frac{3\pi}{2}$ using 6 approximating rectangles and use left and right endpoints.

4. Estimate the area under the curve of $f(x) = \frac{1}{4}x^2 + 3$ from $x = 0$ to $x = 4$ using 4 approximating rectangles and use midpoints.

Evaluate the integrals:

1. $\int (x^7 + x^4 + 2)\, \mathrm{dx}$

2. $\int \frac{r^5 + r^2 - r}{r^2}\, dr$

3. $\int_1^3 (x^3 + 5x + 9)\, \mathrm{dx}$

4. $\int_{-2}^{2} e^3 dx$

Answer Sheet:

Definitions:

1. $A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_1)\,\Delta x + f(x_2)\,\Delta x + \ldots + f(x_n)\,\Delta x]$
2. $\int_a^b f(x)\,dx = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i)\,\Delta x\right)$, $\Delta x = \frac{b-a}{n}$ and x_i is any point in the interval
3. Midpoint Rule: $\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(\overline{x_i})\,\Delta x = \Delta x\,[f(\overline{x_i}) + \ldots + f(\overline{x_i})]$ where $\Delta x = \frac{b-a}{n}$ and $\overline{x_i} = \frac{1}{2}(x_{i-1}, x_i)$

Finding the area under the curve:

1. $\Delta x = \frac{b-a}{2} = \frac{36-0}{6} = 6$
   $f(x_1) = f(0) = 27$
   $f(x_2) = f(6) \approx 26$
   $f(x_3) = f(12) \approx 25$
   $f(x_4) = f(18) \approx 22$
   $f(x_5) = f(24) = 18$
   $f(x_6) = f(30) = 12$
   $f(x_7) = f(36) = 3$

   Left endpoints: $A = (27)(6) + (26)(6) + (25)(6) + (22)(6) + (18)(6) + (12)(6) = 780$
   Right endpoints:
   $A = (26)(6) + (25)(6) + (25)(6) + (22)(6) + (18)(6) + (12)(6) + (18)(6) = 636$
   Midpoints:
   $f(\overline{x_1}) = f(3) \approx 26.5$
   $f(\overline{x_2}) = f(9) \approx 25.5$
   $f(\overline{x_3}) = f(15) \approx 23$
   $f(\overline{x_4}) = f(21) \approx 19.5$
   $f(\overline{x_5}) = f(27) = 15$
   $f(\overline{x_6}) = f(33) \approx 8.5$
   $A = (26.5)(6) + (25.5)(6) + (23)(6) + (19.5)(6) + (15)(6) + (8.5)(6) = 708$

2. $A = \sum_{i=1}^{1} f(x_i)\,\Delta x$
   $\Delta x = \frac{5-1}{4} = 1$

$f(x_1) = f(1) = (1)^2 - 1 = 0$
$f(x_2) = f(2) = (2)^2 - 1 = 3$
$f(x_3) = f(3) = (3)^2 - 1 = 8$
$f(x_4) = f(4) = (4)^2 - 1 = 15$
$f(x_5) = f(5) = (5)^2 - 1 = 24$

Left endpoints: A= (0)(1)+(3)(1)+(8)(1)+(15)(1)= 26

Right endpoints: A= (3)(1)+(8)(1)+(15)(1)+(24)(1)= 50

3. Left endpoint: $\frac{2\pi+\pi\sqrt{2}}{2}$
   Right endpoint: $\frac{\pi\sqrt{2}}{2}$

$$A= \sum_{i=1}^{1} f(x_i)\,\Delta x$$

$$\Delta x = \frac{\frac{3\pi}{2}}{6} = \frac{\pi}{4}$$

$f(x_1) = f(0) = 4sin(0) = 0$
$f(x_2) = f(\frac{\pi}{4}) = 4sin(\frac{\pi}{4}) = 2\sqrt{2}$
$f(x_3) = f(\frac{\pi}{2}) = 4sin(\frac{\pi}{2}) = 4$
$f(x_4) = f(\frac{3\pi}{4}) = 4sin(\frac{3\pi}{4}) = 2\sqrt{2}$
$f(x_5) = f(\pi) = 4sin(\pi) = 0$
$f(x_6) = f(\frac{5\pi}{4}) = 4sin(\frac{5\pi}{4}) = -2\sqrt{2}$
$f(x_7) = f(\frac{3\pi}{2}) = 4sin(\frac{3\pi}{2}) = -4$

Left endpoints:
$A= (2\sqrt{2})(0) + (2\sqrt{2})(\frac{\pi}{4}) + (4)(\frac{\pi}{4}) + (2\sqrt{2})(\frac{\pi}{4}) + (0)(\frac{\pi}{4}) + (-2\sqrt{2})(\frac{\pi}{4}) = \frac{2\pi+\pi\sqrt{2}}{2}$
Right endpoints:
$A= (2\sqrt{2})(\frac{\pi}{4}) + (4)(\frac{\pi}{4}) + (2\sqrt{2})(\frac{\pi}{4}) + (0)(\frac{\pi}{4}) + (-2\sqrt{2})(\frac{\pi}{4}) + (-4)(\frac{\pi}{4}) = \frac{\pi\sqrt{2}}{2}$

4. $\frac{69}{4}$

$f(0.5) = \frac{1}{4}(0.5)^2 + 3 = \frac{49}{16}$
$f(1.5) = \frac{1}{4}(1.5)^2 + 3 = \frac{57}{16}$
$f(2.5) = \frac{1}{4}(2.5)^2 + 3 = \frac{73}{16}$
$f(3.5) = \frac{1}{4}(3.5)^2 + 3 = \frac{97}{16}$

A= $(\frac{49}{16})(1) + (\frac{57}{16})(1) + (\frac{73}{16})(1) + (\frac{97}{16})(1) = \frac{276}{16} = \frac{69}{4}$

Evaluate the integrals:

1. $\frac{x^8}{8} + \frac{x^5}{5} + 2x + C$

   Solution: $\frac{x^{7+1}}{7+1} + \frac{x^{4+1}}{4+1} + \frac{2x^{0+1}}{0+1} + C$
   $\frac{x^8}{8} + \frac{x^5}{5} + 2x + C$

2. $\frac{r^4}{4} + r + ln(r) + C$

   Solution: $\int \frac{r^5+r^2+r}{r^2} = \int \frac{r^5}{r^2} + \frac{r^2}{r^2} + \frac{r}{r^2} = \int r^3 + 1 + \frac{1}{r}$

   $\frac{r^{3+1}}{3+1} + \frac{r^{0+1}}{0+1} + ln(r) + C$, the antiderivative of $\frac{1}{r} = ln(r)$

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   Solution: $\frac{x^{3+1}}{3+1} + \frac{5x^{1+1}}{1+1} + \frac{9x^{0+1}}{0+1} = \frac{x^4}{4} + \frac{5x^2}{2} + 9x \,|^3{}_1$
   $[\frac{(3)^4}{4} + \frac{5(3)^2}{2} + 9(3)] - [\frac{(1)^4}{4} + \frac{5(1)^2}{2} + 9(1)]$
   $= 232$

4. $4e^3$

   Solution: $\int e^3 dx = e^3 x \,|^2{}_{-2}$, remember e^3 is just a constant

   $[e^3(2)] - [e^3(-2)] = 2e^3 + 2e^3$
   $= 4e^3$